ON PEIXOTO'S CONJECTURE FOR FLOWS
ON NON-ORIENTABLE 2-MANIFOLDS

CARLOS GUTIERREZ AND BENITO PIRES

ABSTRACT. Contrary to the case of vector fields on orientable compact 2-manifolds, there is a smooth vector field X on a non-orientable compact 2-manifold with a dense orbit (and therefore without closed orbits) whose phase portrait –up to topological equivalence– remains intact under a one-parameter family of twist perturbations localized in a flow box of X.

This article was published on the Proc. Amer. Math. Soc. 133 (2005), no. 4, 1063–1074 (electronic).

REFERENCES

1. A. Pierre, M. Malkin and E. Zhuzhoma. *On the C^r–closing lemma for surface flows and expansions of points of the circle at infinity.* Preprint IML-2001.
2. S. Aranson and E. Zhuzhoma. *On the C^r-closing lemma and the Koebe-Morse coding of geodesics on surfaces.* Journ. Dyn. Contr. Syst., **7** (2001), No. 1, 15–48. MR **1817328 (2001m:**37042)
3. F. Abdenur, A. Avila, J. Bochi. *Robust transitivity and topological mixing for C^1-flows.* Proc. Amer. Math. Soc. **132** (2004), 699–705. MR 2019946
4. C. R. Carrol. *Rokhlin towers and C^r closing for flows on $\mathbf{C}^2$.* Ergod. Th. & Dynam. Sys. (1992), **12**, 683–706. MR **1200337 (94c:**58177)
5. J. Franks. *Rotation numbers and instability sets.* Bull. Amer. Math. Soc. **40** (2003), 263-279. MR **1978565 (2004h:**37063)
6. C. Gutierrez. *Structural stability for flows on the torus with a cross-cap.* Transactions of the American Mathematical Society, Volume 241 (1978). MR **0492303 (80k:**58065)
7. C. Gutierrez. *On the C^r-Closing for Flows on 2-Manifolds.* Nonlinearity **13** (2000), No. 6, 1883–1888. MR **1794837 (2001m:**37043)
8. C. Gutierrez. *Smoothing continuous flows on two-manifolds and recurrences.* Erg. Th. and Dyn. Sys. (1986), **6**, 17–44. MR **0837974 (87k:**58222)
9. C. Gutierrez. *A counter-example to a C^2- closing lemma.* Erg. Th. and Dyn. Sys. (1987), **7**, 509–530. MR **0922363 (89k:**58240)
10. C. Gutierrez. *Smooth Nonorientable Nontrivial Recurrence on Two-Manifolds.* Journal of Dif. Equations, vol. 29, N^0 3, pp. 338-395, 1978. MR **0507486 (80d:**58059)
11. C. Gutierrez and B. F. Pires. *On a C^r–Connecting Lemma for Flows on a Non-Orientable 2-Manifold.* To be written.
12. C. Gutierrez and J. Sotomayor *An Approximation Theorem for Immersions with Stable Configurations of Lines of Principal Curvature.* Bifurcation, Théorie Ergodique et Applications. Dijon, Juin 1981: Astérisque, 98-99, pp. 195-215, 1982. MR **0730276 (85b:**53002)
13. C. Gutierrez and J. Sotomayor. *Lines of Curvature and Umbilical Points on Surfaces.* 18^0 Colóquio Brasileiro de Matemática, IMPA, 1991.
14. S. Hayashi. *Connecting invariant manifolds and the solution of the C^1 stability and Ω-stability conjecture for flows.* Annals of Mathematics, **145** (1997), 81–137. MR **1432037 (98b:**58096)

15. S. Hayashi. *Correction to Connecting invariant manifolds and the solution of the C^1 stability and Ω-stability conjecture for flows.* Annals of Mathematics, **150** (1999), 353–356. MR **1715329 (2000h:**37029)
16. S. Hayashi. *A C^1 make or break lemma.* Bol. Soc. Brasil. Mat. (N.S.) **31** (2000), no.3, 337–350. MR **1817092 (2002c:**37042)
17. M. Herman. *Exemples de flots hamiltoniens dont aucune perturbation en topologie C^∞ n'a d'orbites périodiques sur un ouvert de surfaces d'énergies.* C. R. Acad. Sci. Paris Sér. I Math. **t. 312** (1991), No. 13, 989–994. MR **1113091 (92j:**58034)
18. M. Herman. *Différentiabilité optimale et contre-exemples à la fermeture en topologie C^∞ des orbites récurrentes de flots hamiltoniens.* C. R. Acad. Sci. Paris Sér. I Math. **t. 313** (1991), No. 1, 49–51. MR **1115947 (92m:**58046)
19. R. Mañé. *An ergodic closing lemma.* Ann. of Math. (2) **116** (1982), No. 3, 503–540. MR **0678479 (84f:**58070)
20. R. Mañé. *On the creation of homoclinic points.* Inst. Hautes Études Sci. Publ. Math. No. 66, (1988), 139–159. MR **0932137 (89e:**58089)
21. N. Markley.*The Poincaré-Bendixson Theorem for the Klein Bottle.* Trans. Amer. Math. Soc. **135** (1969). MR **0234442 (38:**2759)
22. F. Oliveira. *On the generic existence of homoclinic points.* Ergodic Theory and Dynamical Systems **7** (1987) No. 4, 567–595. MR **0922366 (89j:**58104)
23. F. Oliveira. *On the C^∞ genericity of homoclinic orbits.* Nonlinearity **13** No. 3 (2000), 653–662. MR **1758993 (2001h:**37106)
24. M. Peixoto. *Structural stability on two-dimensional manifolds.* Topology, 1 (1962), pp. 101–120. MR **0142859 (26:**426)
25. D. Pixton. *Planar homoclinic points.* Jr. Differential Equations **44** (1982), No. 3, 365–382. MR **0661158 (83h:**58077)
26. C. Pugh. *An improved closing lemma and a general density theorem.* Amer. J. Math. **89** (1967), 1010–1021. MR **0226670 (37:**2257)
27. C. Pugh. *Against the C^2-closing lemma.* J. Differential Equations 17 (1975), 435–443. MR **0368079 (51:**4321)
28. C. Pugh. *A special C^r-closing lemma.* Geometric Dynamics (Rio de Janeiro, 1981), 636–650, Lecture Notes in Math., **1007**, Springer, Berlin, 1983. MR **0730291 (85e:**58120)
29. C. Pugh. *The C^1 connecting lemma.* J. Dynam. Differential Equations 4 (1992), no. 4, 545–553. MR **1187222 (93i:**58091)
30. C. Pugh and C. Robinson. *The C^1-closing lemma, including Hamiltonians.* Ergodic Th. and Dynamical Systems **3** (1983), No. 2, 261–313. MR **0742228 (85m:**58106)
31. C. Bonatti and S. Crovisier. *Récurrence et généricité.* Preprint (2003), no. 324, Institute de Mathématiques de Bourgone. MR **1990025 (2004d:**37028)
32. C. Robinson *Closing stable and unstable manifolds on the two sphere.* Proc. Amer. Math. Soc. 41 (1973), 299–303. MR **0321141 (47:**9674)
33. F. Takens. *Homoclinic points in conservative systems.* Invent. Math. **18** (1972), 267–292. MR **0331435 (48:**9768)
34. L. Wen and Z. Xia. *C^1 connecting lemmas.* Trans. Amer. Math. Soc. **352** (2000), no. 11, 5213–5230. MR **1694382 (2001b:**37024)

DEPARTAMENTO DE MATEMÁTICA, INSTITITUO DE CIÊNCIAS MATEMÁTICAS E DE COMPUTAÇÂO, UNIVERSIDADE DE SÃO PAULO, AV. DO TRABALHADOR SÃO CARLENSE, 400, CENTRO, CEP 13560-970 SÃO CARLOS - SP, BRAZIL
  E-mail address: gutp@icmc.usp.br

DEPARTAMENTO DE MATEMÁTICA, INSTITITUO DE CIÊNCIAS MATEMÁTICAS E DE COMPUTAÇÂO, UNIVERSIDADE DE SÃO PAULO, AV. DO TRABALHADOR SÃO CARLENSE, 400, CENTRO, CEP 13560-970 SÃO CARLOS - SP, BRAZIL
  E-mail address: bpires@icmc.usp.br